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Stochastic Availability of a Serial Solar Photovoltaic System

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Abstract. *The prospects for modeling and decision-making regarding maintenance in solar photovoltaic systems are covered in the current research. In reliability theory, redundancy is a technique widely used to improve system availability and reliability. The k-out-of-n : G system as a popular type of redundancy is often encountered in industrial systems. Evaluation of a simulation model that will be created for the solar photovoltaic system will be used to identify these opportunities. We report on simulation of availability and decision-making using a newly constructed model. Four subsystems namely, solar panel, charge controller, batteries and inverter subsystem, make up the solar photovoltaic current system, with two conceivable states: functional and failing. A Markov-based probabilistic simulated model has been created while taking a few presumptions into account. Each subsystem's availability decision matrix is also created to offer several levels of availability. The performance or availability of each solar photovoltaic subsystem is examined in light of this analysis, and maintenance priorities are then established for the current system. Through the schematic diagram of state of the system, availability model is formulated using mnemonic rule and Chapman - Kolmogorov differential difference equations are developed and solved by Runge–Kutta method of order four.*

Keywords: Modeling, Solar photovoltaic; Markov-modeling, Subsystems; Maintenance; k-out-of-n : G system.

AMS Subject Classifications : 65S05, 65T60, 92B05, 92D30

1. Introduction

Engineering maintenance is crucial for maintaining the availability of a plant. The availability of a solar photovoltaic system plant is higher the better the reliability and maintainability. According to Van et al., [1], there can be two ways that specific pieces of equipment can affect reliability when it comes to the overall problem of equipment failure and maintenance. Either there is a loss of production owing to frequent downtime, or to equipment only occasional malfunctions is so sophisticated or crucial as to take a very long time to replace or repair.

Several mathematical models that handle a wide range of complexities have been described in literature for the prediction of availability. Among which is Engineering Reliability into the Design and Operation of Systems, by Dhillon [2]. The failure and repair rates are considered to be constant in the majority of these models. In other words, the times between breakdown and repair are exponentially distributed. Models for performance, availability, and performability were designed for reliability. Different modeling paradigms, including fault trees, Markov chains, Petri nets, and activity nets, are covered by these tools. Markov chains have the capacity to model systems with shared repair, which is advantageous. These can model a dynamic system structure with greater accuracy than fault trees or reliability block diagrams can, Malhotra and Trivedi [3]. EHARP is one of the techniques for Markov analysis. Phased mission reliability analysis for systems with variable configuration is shown to be computationally efficient by Ritcey et al. [4]. System Reliability Computation with Latent Failures and Monitoring, was reported by Somani et al. [5]. In ACM Computing Surveys, Survey of Software Tools for Evaluating Reliability, Availability, and Serviceability was analyzed by Johnson and Malek [6]. The majority of industrial and manufacturing systems have been modeled using the Markov birth-death process to determine their availability under the premise that the failure and repair rates of each subsystem follow an exponential distribution, Aggarwal et al. in [7] and Kumar et al. [8]. A discussion of how solar photovoltaic (PV) modules can increase the capacity of solar distillation units to produce water sustainably was provided by Sharon et al. [17]. A k-out-of-n: G system, we employ in this work, has n components, all of which are operational at startup even though only k of them are necessary for the system to function normally. The cables in a suspension bridge, the multi-pump system in a hydraulic control system, the multi display system in a cockpit, and the multi-engine system in an aircraft are classic examples of its applications. Other examples include communication systems with multiple transmitters, power transmission and distribution systems, electronic circuit design, and cable systems in suspension bridges. When a k-out-of-n: G system is considered for modeling, exponential distributions of random times involved are usually assumed, for example, see e.g. Moustafa [23]

1.1. **Organization of the paper**

Reducing carbon dioxide emissions from conventional power generation is the main driver behind our deployment of solar energy. Additionally, the breakdown of industrial machinery due to electricity fluctuations has resulted in very little advancement in technical growth worldwide. The reliability, availability, and cost of maintaining distribution networks with PV will therefore be investigated. This will call for approaches and tools to quantify the reliability of grid-connected PV systems.

Researchers have made significant contributions to improving the effectiveness and performance of some photovoltaic systems as well as by examining the elements that limit their effectiveness. The strength, efficiency, and performance of the Photovoltaic (PV) system are assessed using availability metrics, which are not well understood. More research is needed on the effectiveness of testing the strength and performance enhancement of PV systems. The present paper introduces an availability modeling approach to examine the overall performance of the PV system due to the large amount of data from the PV system. In this study, we present a brand-new PV system model with four subsystems: the panel, the inverter, the battery bank, and the control charger. To determine the availability characteristics for each subsystem, a system of first order ordinary differential equations is formed using the transition diagram and solved recursively using the supplemental variable technique. The objective of this study is to create dependability models for assessing the PV system's strength. The findings of this research will be useful to home and commercial plant management, businesses aiming to adopt PV as their electricity and energy source, and manufacturing processes.

The description of the solar photovoltaic system utilized to create the transition diagram is presented and discussed in Section 2. This section also includes a list of the hypotheses that were utilized to create the simulation model. The construction of the simulation model is discussed in Section 3, along with a brief overview of the Markov technique. The performance assessment for decision-making in this study is described in Section 4. The results and conclusions are respectively described in Sections 5 and 6.

2. The Solar Photovoltaic System

Over the past ten years, solar energy has become increasingly used as a source of heat and electricity. Particularly, photovoltaic (PV) systems, which currently account for more than 10% of the electricity energy supply, have established a crucial role in the electrical energy sectors. The linked PV % growth is due to a variety of factors. such as affordable installation costs, quick energy return, and investment payback that includes potential customer stimulation, Maihulla et al. [9]. The essential parts of a solar PV system are the PV modules, controller, batteries, and inverter. Every component is set up in a series configuration, Anas and Ibrahim [10]. Reliability, availability, maintainability, and dependability (RAMD) analyses have been used by [10] to evaluating the strength of the system at components level. The research conducted by Anas and Yusuf [11] provided a transition diagram of each subsystem of the solar photovoltaic, and

Chapman-Kolmogorov differential equations are built using the Markov birth-death process for each variable in each subsystem. Among their conclusion is that it is statistically independent for both random failure and repair time variables to have an exponential distribution. With the device, a successful repair facility is still accessible. In all the aforementioned researches, none developed a systematic diagram utilizing the mnemonic rule to create Chapman-Kolmogorov differential difference equations, which were then resolved using the order four Runge-Kutta method. This has prompted our research together with the significance of the method in solving the system of first order differential equations. Three of the five panels were thought to be active at once. It was presumed that 1 charge controller out of 1 was working. For the system to function, it was believed that 2 out of 3 batteries would be in an operational state. Finally, for the system to be in an operational state, 1 out of 1 inverter is required. The system pictorial representation is illustrated in Figure 1 below.

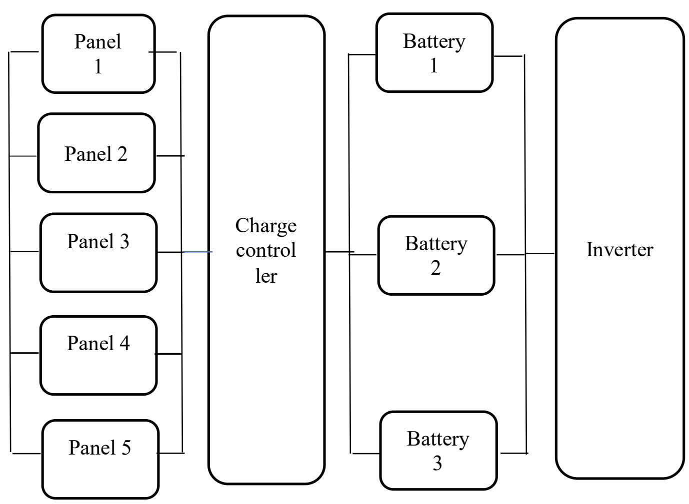
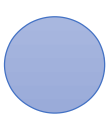
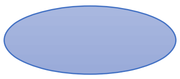


Figure 1: System block diagram.

2.1. **Notations**

 A sign that the system is fully operational.

 A sign that the system is operating at a lower capacity

 Shown to be a failure state indicator for the system.

$P_0(t)$ is the probability for the system is in full working state.

$P_1(t)$, $P_2(t)$, $P_3(t)$, $P_4(t)$, and $P_5(t)$, are the probability for the system is in reduced capacity.

$P_6(t)$ to $P_{22}(t)$ are probability that the system is in complete failure state.

$\frac{d}{dt}P_i(t)$ Represent the derivatives with respect to time t.

$A_T(\infty)$ Steady state availability of the system

$P_0(\infty)$ Probability of full working state when the system is new, and is determine using the condition normalizing:

$P_0(\infty)+P_1(\infty)+\ldots+P_{22}(\infty)=1,$

$\delta_1, \delta_2, \delta_3, \delta_4$ are the failure rates for the subsystem1, 2, 3 and 4 respectively.

$\eta_1, \eta_2, \eta_3, \eta_4$ are the repair rates for the subsystem1, 2, 3 and 4 respectively.

3. Modeling for Availability Simulation

The foundation of Markov modeling hosts the idea that a system and its parts can exist in several states. At the most basic level, a component can be either up or down, whereas a system can be in any specified state, based on the components that make up the system and on the state they are in. A Markov model, often known as a state-space model, explains how states change over time. States that the transition probabilities depend on are only the present state of the system as stated by Wolstenholme [18]. All components, the possible states they could be in, and the frequency at which they can change states should be included in the model. A transition diagram that is based on ideas provided by Kumar et al. [19] describes the flow of states for the system under study, as shown in figure 2. This is a logical representation of all state probabilities encountered throughout the solar photovoltaic system failure analysis. When a component fails,

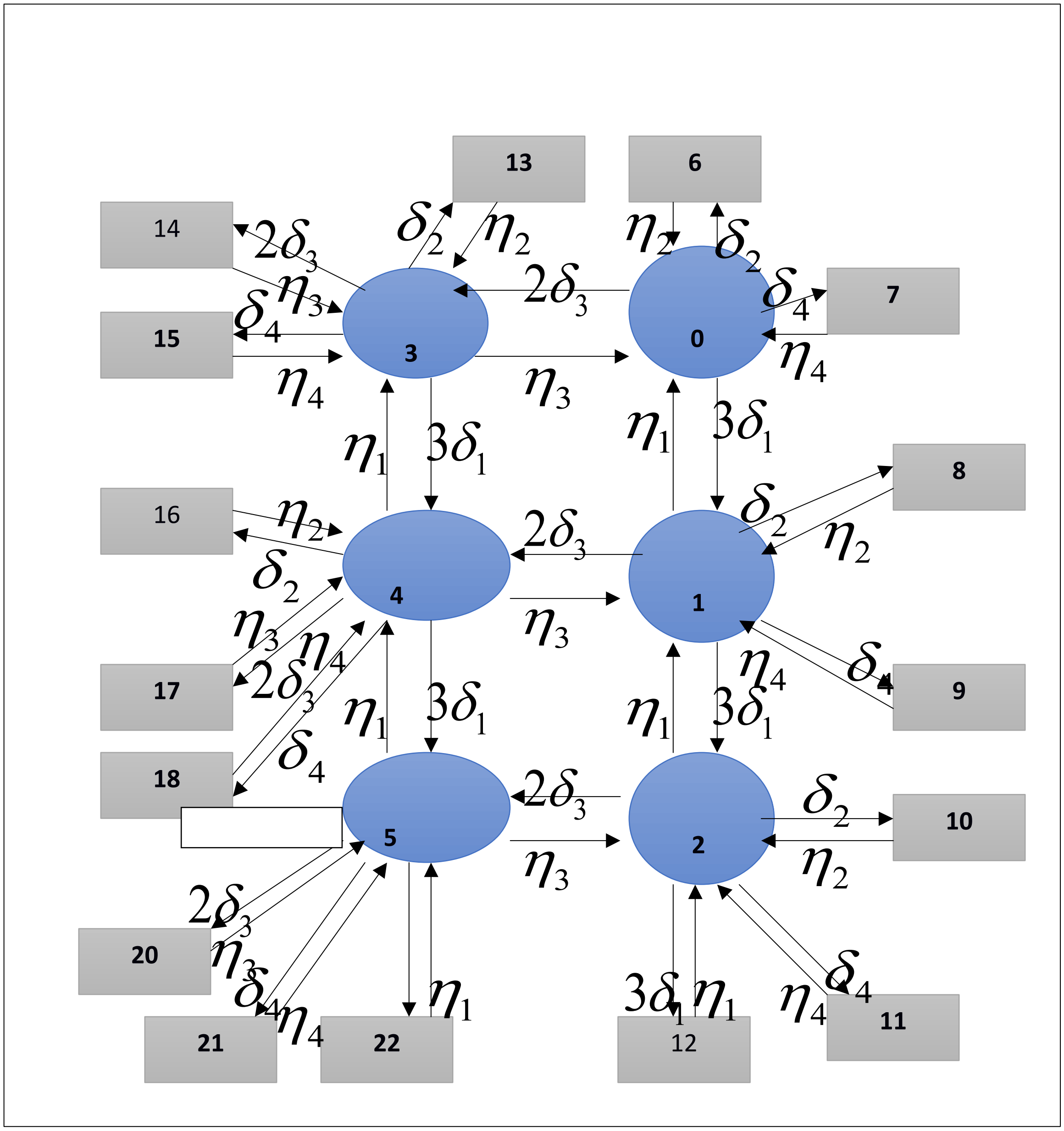


Figure 2: System transition diagram.

it frequently shifts from its up state to its down state, and when it is repaired, it frequently returns to its up state, as studied by Conner [20]. The model's basic input data are the failure and repair rates of the various subsystems. The formulated joint probability functions are based on the transition diagram are used to formulate. These probabilities offer the opportunity to use the Markovian approach for the availability analysis of the power generation process because they are mutually exclusive. In this respect, the Reliability and performance prediction of a small serial solar photovoltaic system for rural consumption using the Gumbel-Hougaard family copula was analyzed by Anas and Yusuf [22].

3.1. Assumptions

The probabilistic model has been developed using the following presumptions:

1. There is no such thing as simultaneous failure, (Khanduja et al. [12]).
2. At any one time, the system is either fully operational or failing, (Gupta et al. [13]).
3. Repair and/or replacement are included in service.
4. The nature and capabilities of standby subsystems are identical to those of active systems.
5. There are adequate repair facilities, (Srinath [14]).
6. Failure/repair rates are stable and statistically independent over time, (Kumar et al. [15]).
7. Repair and system failure follow an exponential distribution.
8. For a certain period of time, a repaired system performs at par with a brand-new one, (Gupta et al. [16]).

3.2. Development of models

The system's transition diagram in Figures 1 and 2 is represented by the differential difference equations shown below, which are generated using the Markov birth-death process. As the methodology adopted by Yusuf et al. [21]. Where (1) to (5) are operational, and (6) to (22) are failure states.

$$\left(\frac{d}{dt}+3\delta_1+\delta_2+2\delta_3+\delta_3\right)p_0(t)=\eta_1 p_1(t)+\eta_3 p_3(t)+\eta_2 p_6(t)+\eta_4 p_7(t) \tag{1}$$

$$\left(\frac{d}{dt}+\eta_1+3\delta_1+\delta_2+2\delta_3+\delta_4\right)p_1(t)=3\delta_1 p_0(t)+\eta_1 p_2(t)+\eta_3 p_4(t)+\eta_2 p_8(t)+\eta_4 p_9(t) \tag{2}$$

$$\left(\frac{d}{dt}+\eta_1+3\delta_1+\delta_2+2\delta_3+\delta_4\right)p_2(t)=3\delta_1 p_1(t)+\eta_3 p_5(t)+\eta_2 p_{10}(t)+\eta_4 p_{11}(t)+\eta_1 p_{12}(t) \tag{3}$$

$$\left(\frac{d}{dt}+\eta_1+3\delta_1+\delta_2+2\delta_3+\delta_4\right)p_3(t)=2\delta_3 p_0(t)+\eta_1 p_4(t)+\eta_2 p_{13}(t)+\eta_3 p_{14}(t)+\eta_4 p_{15}(t) \tag{4}$$

$$\left(\frac{d}{dt}+\eta_1+\eta_3+3\delta_1+\delta_2+2\delta_3+\delta_4\right)p_5(t)=2\delta_3 p_2(t)+3\delta_1 p_4(t)+\eta_2 p_{19}(t)+\eta_3 p_{20}(t)+\eta_4 p_{21}(t)+\eta_1 p_{22}(t) \tag{5}$$

$$\left(\frac{d}{dt}+\eta_2\right)p_6(t)=\delta_2 p_0(t) \tag{6}$$

$$\left(\frac{d}{dt}+\eta_4\right)p_7(t)=\delta_4 p_0(t) \tag{7}$$

$$\left(\frac{d}{dt}+\eta_2\right)p_8(t)=\delta_2 p_1(t) \tag{8}$$

$$\left(\frac{d}{dt}+\eta_4\right)p_9(t)=\delta_4 p_1(t) \tag{9}$$

$$\left(\frac{d}{dt}+\eta_2\right)p_{10}(t)=\delta_2 p_2(t) \tag{10}$$

$$\left(\frac{d}{dt}+\eta_4\right)p_{11}(t)=\delta_4 p_2(t) \tag{11}$$

$$\left(\frac{d}{dt}+\eta_1\right)p_{12}(t)=3\delta_1 p_2(t) \tag{12}$$

$$\left(\frac{d}{dt}+\eta_2\right)p_{13}(t)=\delta_2 p_3(t) \tag{13}$$

$$\left(\frac{d}{dt}+\eta_3\right)p_{14}(t)=2\delta_3 p_3(t) \tag{14}$$

$$\left(\frac{d}{dt}+\eta_4\right)p_{15}(t)=\delta_4 p_3(t) \tag{15}$$

$$\left(\frac{d}{dt}+\eta_2\right)p_{16}(t)=\delta_2 p_4(t) \tag{16}$$

$$\left(\frac{d}{dt}+\eta_3\right)p_{17}(t)=2\delta_3 p_4(t) \tag{17}$$

$$\left(\frac{d}{dt}+\eta_4\right)p_{18}(t)=\delta_4 p_4(t) \tag{18}$$

$$\left(\frac{d}{dt}+\eta_2\right)p_{19}(t)=\delta_2 p_5(t) \tag{19}$$

$$\left(\frac{d}{dt}+\eta_3\right)p_{20}(t)=2\delta_3 p_5(t) \tag{20}$$

$$\left(\frac{d}{dt}+\eta_4\right)p_{21}(t)=\delta_4 p_5(t) \tag{21}$$

$$\left(\frac{d}{dt}+\eta_1\right)p_{22}(t)=3\delta_1 p_5(t) \tag{22}$$

with initial condition

$$p_i(t)=\begin{cases}1, & i=0\\ 0, & i=1,2,3,\ldots,22\end{cases} \tag{23}$$

4. Result and Discussion

Using the failure and repair rates, the following were obtained using Yusuf et al. [21].

The Matlab software were used in driving the analytical solution for each probability state, as in Table 1 below.

Table 1: Values of probability states in terms of $P_0(t)$

$p_1(t)=k_1p_0(t)$	$p_9(t)=k_1y_1p_0(t)$	$p_{17}(t)=2k_4y_3p_0(t)$
$p_2(t)=k_2p_0(t)$	$p_{10}(t)=k_2y_2p_0(t)$	$p_{18}(t)=k_4y_4p_0(t)$

$p_3(t) = k_3 p_0(t)$	$p_{11}(t) = k_1 y_4 p_0(t)$	$p_{19}(t) = k_5 y_2 p_0(t)$
$p_4(t) = k_4 p_0(t)$	$p_{12}(t) = 3k_2 y_1 p_0(t)$	$p_{20}(t) = 2k_5 y_3 p_0(t)$
$p_5(t) = k_5 p_0(t)$	$p_{13}(t) = k_3 y_2 p_0(t)$	$p_{21}(t) = k_5 y_4 p_0(t)$
$p_6(t) = y_2 p_0(t)$	$p_{14}(t) = 2k_3 y_3 p_0(t)$	$p_{22}(t) = 3k_5 y_2 p_0(t)$
$p_7(t) = y_4 p_0(t)$	$p_{15}(t) = k_3 y_4 p_0(t)$	
$p_8(t) = k_1 y_2 p_0(t)$	$p_{16}(t) = k_4 y_2 p_0(t)$	

where

$$y_m = \frac{\eta_m}{\delta_m},\ m = 1, 2, 3, 4 \tag{24}$$

and the constants k_1 to k_5 are:

$$k_1 = \frac{3\delta_1 N_{11}}{D_{11}},\ k_2 = \frac{9\delta_1^2 N_{22}}{D_{22}},\ k_3 = \frac{2\delta_3 N_{33}}{D_{11}},\ k_4 = \frac{6\delta_1\delta_3 N_{44}}{D_{11}},\ k_5 = \frac{18\delta_1^2 N_{55}}{D_{22}} \tag{25}$$

with

$$N_{11} = 9\delta_1^2\eta_3 + \left(3\eta_1\eta_2 + 3(\eta_2+\eta_3)(2\delta_3+\eta_3)\right)\delta_1 + \eta_1\eta_3(4\delta_3+\eta_2+\eta_3) + \eta_3(2\delta_3+\eta_3)(2\delta_3+\eta_2) \tag{26}$$

$$N_{22} = 9\delta_1^2\eta_3 + \left(3\eta_1\eta_2 + 3\eta_3(4\delta_3+\eta_2+\eta_3)\right)\delta_1 + \eta_1^2\eta_3 + \eta_1\eta_3(4\delta_3+\eta_2+\eta_3) + \eta_3\left(4\delta_3^2 + 4\delta_3\eta_3 + \eta_2\eta_3\right) \tag{27}$$

$$\begin{aligned} N_{33} = {} & 9\delta_1^2\eta_1 + \left(3\eta_1^2 + \eta_1(12\delta_3+6\eta_3) + 6\delta_1\delta_3(\eta_2-\eta_3)\right)\delta_1 + \eta_1^3 + \eta_1^2 + \eta_1\left(4\delta_3^2 + 4\delta_3\eta_2 + \eta_2\eta_3\right) \\ & + 2\delta_3(\eta_2-\eta_3)(2\delta_3+\eta_3) \end{aligned} \tag{28}$$

$$N_{44} = 9\delta_1^2\eta_3 + (12\delta_3 + 3\eta_1 + 6\eta_3)\delta_1 + \eta_1^2 + \eta_1(4\delta_3+2\eta_3) + (2\delta_3+\eta_3)^2 \tag{29}$$

$$N_{55} = 9\delta_1^2\eta_3 + (12\delta_3 + 3\eta_1 + 3\eta_2 + 3\eta_3)\delta_1 + \eta_1^2 + \eta_1(4\delta_3+2\eta_3) + 4\delta_3^2 + 4\delta_3\eta_3 + \eta_2\eta_3 \tag{30}$$

$$\begin{aligned} D_{11} = {} & \left(9\eta_1\eta_3 + 18\delta_3(\eta_2-\eta_3)\right)\delta_1^2 + \left(3\eta_1^2\eta_2 + \eta_1\left(12\eta_2\delta_3 + 3\eta_2\eta_3 + 3\eta_3^2\right) + 12\delta_3(\eta_2-\eta_3)(\eta_3+\delta_3)\right)\delta_1 + \eta_1^3\eta_3 + \\ & \eta_1^2\eta_3(4\delta_3+\eta_2+\eta_3) + \eta_1\eta_3\left(4\delta_3^2 + 4\delta_3\eta_2 + \eta_2\eta_3\right) + 2\delta_3\eta_3(\eta_2-\eta_3)(2\delta_3+\eta_3) \end{aligned} \tag{31}$$

$$D_{22} = \left(9\eta_1^2\eta_3 + 18\delta_3\eta_1(\eta_2 - \eta_3)\right)\delta_1^2 + \delta_1\left(3\eta_1^2\eta_2 + \eta_1^2\left(12\delta_3\eta_2 + 3\eta_2\eta_3 + 3\eta_3^2\right) + 12\delta_3\eta_1(\eta_2 - \eta_3)(\delta_3 + \eta_3)\right) +$$
$$\eta_1^4\eta_3 + \eta_1^3\eta_3(4\delta_3 + \eta_2 + \eta_3) + \eta_1^2\eta_3\left(4\delta_3^2 + 4\delta_3\eta_2 + \eta_2\eta_3\right) + 2\delta_3\eta_1\eta_3(\eta_2 - \eta_3)(2\delta_3 + \eta_3) \tag{32}$$

By Runge–Kutta method of order four, the Probability of full working state, when the system is new, is presented in equation (33) below.

$$p_0(\infty) = \left(\left(1 + \sum_{i=1}^{5} k_i\right)(1 + y_2) + y_4\left(1 + \sum_{i=2}^{5} k_i\right) + y_1(k_1 + 3k_2 + 3k_5) + 2y_3(k_3 + k_4 + k_5)\right)^{-1} \tag{33}$$

Thus, the Steady state availability of the system follows below in (34).

$$A_T(\infty) = \frac{\left(1 + \sum_{i=1}^{5} k_i\right)}{\left(1 + \sum_{i=1}^{5} k_i\right)(1 + y_2) + y_4\left(1 + \sum_{i=2}^{5} k_i\right) + y_1(k_1 + 3k_2 + 3k_5) + 2y_3(k_3 + k_4 + k_5)} \tag{34}$$

Table 2: Failure rate against repair rate for subsystem 1.

	δ_1=0.00	δ_1=0.00	δ_1=0.00	δ_1=0.00	δ_1=0.00	δ_1=0.00	δ_1=0.00	δ_1=0.00	δ_1=0.00	δ_1=0.01
η_1=0.0	0.9923	0.9924	0.9924	0.9924	0.9924	0.9924	0.9925	0.9925	0.9925	0.9925
η_1=0.0	0.9898	0.9903	0.9906	0.9909	0.9911	0.9913	0.9914	0.9916	0.9917	0.9918
η_1=0.0	0.9874	0.9882	0.9889	0.9894	0.9898	0.9902	0.9904	0.9907	0.9909	0.9910
η_1=0.0	0.9849	0.9862	0.9871	0.9879	0.9885	0.9890	0.9895	0.9898	0.9901	0.9903
η_1=0.0	0.9824	0.9841	0.9854	0.9864	0.9872	0.9879	0.9885	0.9889	0.9893	0.9896
η_1=0.0	0.9800	0.9821	0.9837	0.9849	0.9860	0.9868	0.9875	0.9880	0.9885	0.9889
η_1=0.0	0.9775	0.9800	0.9819	0.9835	0.9847	0.9857	0.9865	0.9871	0.9877	0.9882
η_1=0.0	0.9751	0.9780	0.9802	0.9820	0.9834	0.9845	0.9855	0.9863	0.9869	0.9875
η_1=0.0	0.9727	0.9760	0.9785	0.9805	0.9821	0.9834	0.9845	0.9854	0.9861	0.9868
η_1=0.1	0.9703	0.9740	0.9768	0.9790	0.9808	0.9823	0.9835	0.9845	0.9853	0.9861

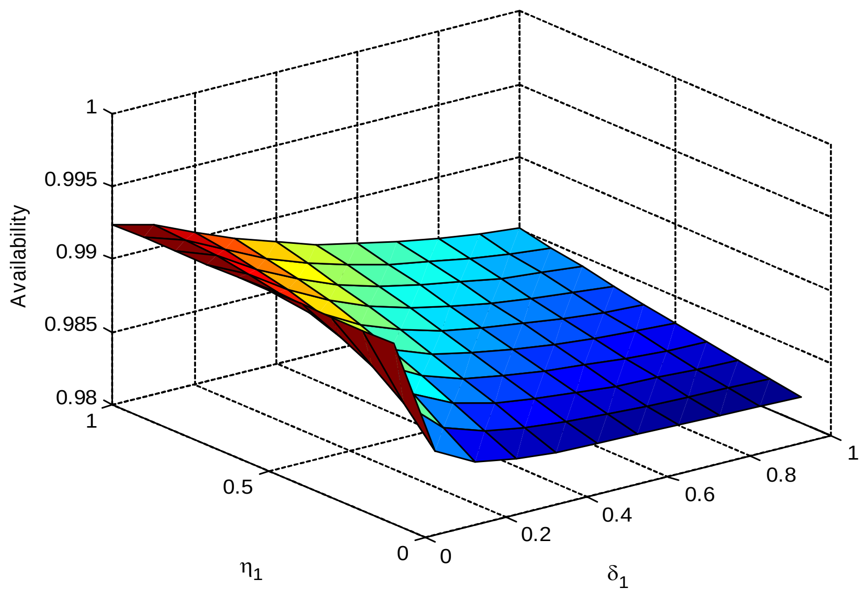


Figure 3: Availability plot for subsystem 1.

Substituting the values of failure and repair rates into (26) to (32), and the values of k_1 through k_5 into the (34), table 1 to table 5 were obtained. The tables were then transformed into the corresponding graphs using Matlab software.

Table 2 and figure 3 showed how the availability of subsystem A was affected by the failure and repair rates for various values of the parameters η_1 and δ_1. As may be seen at the top of the table, the failure and repair rates of other subsystems are maintained constant. Table 2 and Figure 3 make it clear that the availability displayed an increasing pattern in relation to the repair rate η_1 and a decreasing pattern in relation to the failure rate δ_1. It is evident that greater values of η_1 result, the higher system availability, whereas higher values of δ_1 result in lesser availability.

Table 3: Failure rate against repair rate for subsystem 2.

	δ_2=0.00	δ_2=0.00	δ_2=0.00	δ_2=0.00	δ_2=0.00	δ_2=0.00	δ_2=0.00	δ_2=0.00	δ_2=0.00	δ_2=0.01
η_2=0.0	0.9404	0.9772	0.9849	0.9882	0.9901	0.9913	0.9921	0.9927	0.9932	0.9935
η_2=0.0	0.8982	0.9615	0.9753	0.9813	0.9847	0.9868	0.9883	0.9894	0.9903	0.9910
η_2=0.0	0.8596	0.9464	0.9659	0.9745	0.9793	0.9824	0.9846	0.9862	0.9874	0.9884
η_2=0.0	0.8242	0.9317	0.9566	0.9677	0.9740	0.9781	0.9809	0.9830	0.9845	0.9858
η_2=0.0	0.7916	0.9174	0.9476	0.9611	0.9688	0.9737	0.9772	0.9797	0.9817	0.9833
η_2=0.0	0.7614	0.9036	0.9387	0.9545	0.9636	0.9694	0.9735	0.9766	0.9789	0.9807
η_2=0.0	0.7335	0.8902	0.9299	0.9481	0.9585	0.9652	0.9699	0.9734	0.9761	0.9782
η_2=0.0	0.7075	0.8772	0.9214	0.9417	0.9534	0.9610	0.9663	0.9702	0.9733	0.9757
η_2=0.0	0.6834	0.8645	0.9130	0.9354	0.9484	0.9568	0.9627	0.9671	0.9705	0.9732
η_2=0.1	0.6608	0.8523	0.9047	0.9292	0.9434	0.9526	0.9592	0.9640	0.9677	0.9707

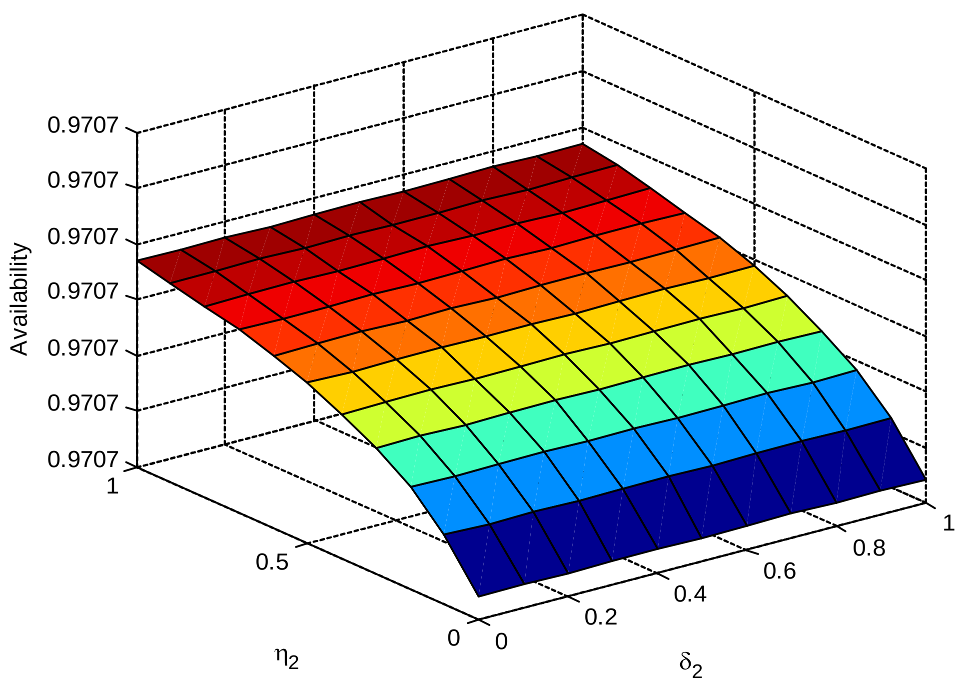


Figure 4: Availability plot for subsystem 4.

Table 3 and picture 4 demonstrated the relationship between the failure and repair rates for various values of the parameters η_2 and δ_2 and the availability of subsystem B. The failure and repair rates of other subsystems are kept constant, as can be seen at the top of the table. It is evident from Table 3 and Figure 4 that the availability exhibited an upward trend in relation to the repair rate and a downward trend in relation to the failure rate. It is obvious that higher values of η_2 lead to increased system availability, whereas higher values of δ_2 lead to worse availability.

Table 4: Failure rate against repair rate for subsystem 3.

	δ_3=0.00	δ_3=0.00	δ_3=0.00	δ_3=0.00	δ_3=0.00	δ_3=0.00	δ_3=0.00	δ_3=0.00	δ_3=0.00	δ_3=0.01
η_3=0.0	0.9923	0.9924	0.9924	0.9924	0.9924	0.9924	0.9924	0.9924	0.9924	0.9924
η_3=0.0	0.9923	0.9923	0.9924	0.9924	0.9924	0.9924	0.9924	0.9924	0.9924	0.9924
η_3=0.0	0.9922	0.9923	0.9923	0.9924	0.9924	0.9924	0.9924	0.9924	0.9924	0.9924
η_3=0.0	0.9921	0.9923	0.9923	0.9923	0.9924	0.9924	0.9924	0.9924	0.9924	0.9924
η_3=0.0	0.9920	0.9922	0.9923	0.9923	0.9923	0.9924	0.9924	0.9924	0.9924	0.9924
η_3=0.0	0.9919	0.9922	0.9923	0.9923	0.9923	0.9923	0.9924	0.9924	0.9924	0.9924
η_3=0.0	0.9918	0.9921	0.9922	0.9923	0.9923	0.9923	0.9923	0.9924	0.9924	0.9924
η_3=0.0	0.9917	0.9921	0.9922	0.9923	0.9923	0.9923	0.9923	0.9923	0.9924	0.9924
η_3=0.0	0.9916	0.9920	0.9922	0.9922	0.9923	0.9923	0.9923	0.9923	0.9923	0.9924
η_3=0.1	0.9915	0.9920	0.9921	0.9922	0.9923	0.9923	0.9923	0.9923	0.9923	0.9923

The association between the failure and repair rates for various values of the parameters η_3 and δ_3 and the availability of subsystem B was shown in Table 4 and figure 5. As noted at the top of the table, the failure and repair rates of other subsystems are maintained constant. Table 4 and Figure 5 make it clear that there was an upward trend in the availability relative to the repair rate and a downward trend relative to the failure rate. It should go without saying that greater values of η_3 result in better availability for the system, whereas larger values of δ_3 result in lower availability.

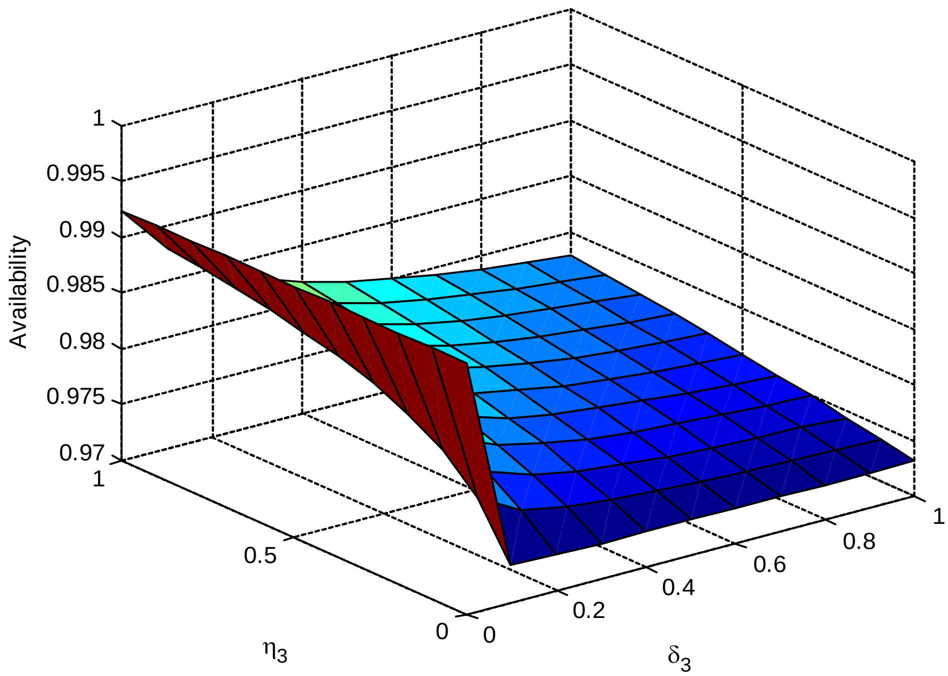


Figure 5: Availability plot for subsystem 3.

Table 5: Failure rate against repair rate for subsystem 4.

	$\delta_4=0.00$	$\delta_4=0.00$	$\delta_4=0.00$	$\delta_4=0.00$	$\delta_4=0.00$	$\delta_4=0.00$	$\delta_4=0.00$	$\delta_4=0.00$	$\delta_4=0.00$	$\delta_4=0.0$
$\eta_4=0.0$	0.9859	0.9908	0.9924	0.9932	0.9937	0.9941	0.9943	0.9945	0.9946	0.9947
$\eta_4=0.0$	0.9764	0.9859	0.9892	0.9908	0.9918	0.9924	0.9929	0.9932	0.9935	0.9937
$\eta_4=0.0$	0.9670	0.9811	0.9859	0.9884	0.9898	0.9908	0.9915	0.9920	0.9924	0.9928
$\eta_4=0.0$	0.9578	0.9764	0.9827	0.9859	0.9879	0.9892	0.9901	0.9908	0.9913	0.9918
$\eta_4=0.0$	0.9488	0.9717	0.9795	0.9835	0.9859	0.9876	0.9887	0.9896	0.9903	0.9908
$\eta_4=0.0$	0.9399	0.9670	0.9764	0.9811	0.9840	0.9859	0.9873	0.9884	0.9892	0.9898
$\eta_4=0.0$	0.9312	0.9624	0.9732	0.9788	0.9821	0.9843	0.9859	0.9872	0.9881	0.9889
$\eta_4=0.0$	0.9226	0.9578	0.9701	0.9764	0.9802	0.9827	0.9846	0.9859	0.9870	0.9879
$\eta_4=0.0$	0.9143	0.9532	0.9670	0.9740	0.9783	0.9811	0.9832	0.9847	0.9859	0.9869
$\eta_4=0.1$	0.9060	0.9488	0.9639	0.9717	0.9764	0.9795	0.9818	0.9835	0.9849	0.9859

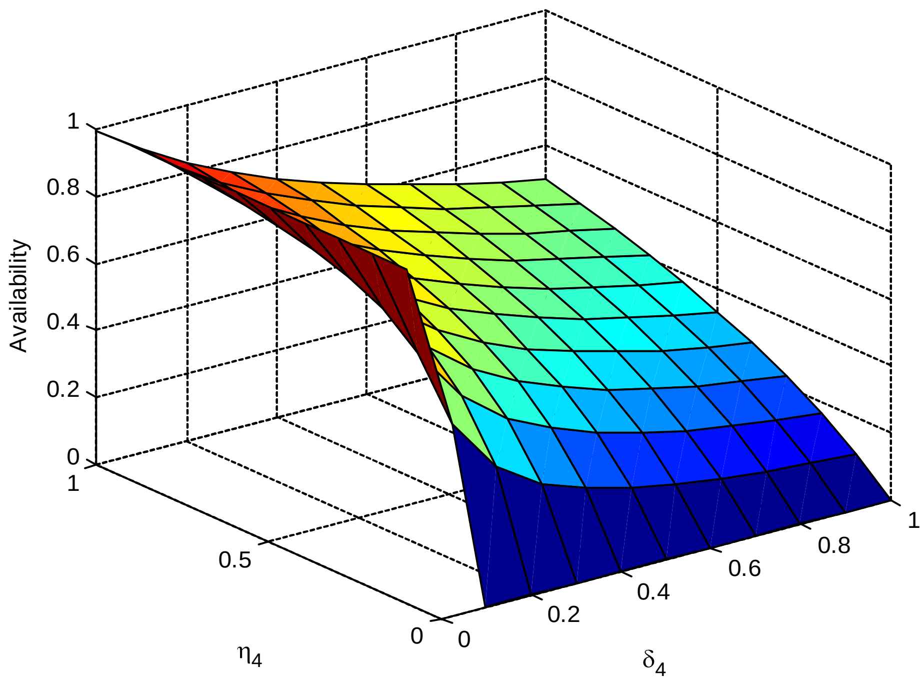


Figure 6: Availability plot for subsystem 4.

Table 5 and figure 6 depict the correlation between the failure and repair rates for various values of the parameters η_4 and δ_4 and the availability of subsystem B. The failure and repair rates of other subsystems are kept constant, as stated at the top of the table. There was an upward trend in the availability relative to the repair rate and a downward trend relative to the failure rate, as shown by Table 5 and Figure 6. It should be obvious that higher values of η_4 lead to better system availability, whereas higher values of δ_4 lead to worse availability.

5. Conclusion

In order to analyze the availability analysis of the system, a series-parallel system configuration with 4 subsystems was built. The system's steady-state availability, busy period, and availability function are all deduced with explicit expressions. The impact of both failure and repair rates on availability has been demonstrated through the use of numerical results. It is clear from the analysis that the following can improve availability:

I. Planting for proper maintenance to prevent catastrophic failure.
II. Maintaining the maximum level of system availability.
III. Adding more redundant, fault-tolerant components or units.

It is clear from the magnitude of the correlation coefficient given in the study that subsystem B is the most important and that its failure would be disastrous for the entire system.

With some adjustments and suppositions, the model presented in this paper will help management prevent inaccurate reliability assessments and subsequent improper decisions that could result in wasted spending. The current task can be expanded to include monitoring for failure dependability and condition, allowing management to decide on the best maintenance and replacement.

References

[1] A. K. Somani, J. A. Ritcey, and S. H. Au, Computationally efficient phased mission reliability analysis for sstems with variable configuration, *IEEE Transactions on Reliability* **41**(4), (1992), 504–511.

[2] A. K. Somani, T. Sharma, and P. H. Nguyen, Reliability computation of systems with latent-failures and monitoring,*Proceedings of IEEE 1994 Annual Reliability and Maintainability Symposium*, (1994), 195–200.

[3] A. M. Johnson Jr. and M. Malek, Survey of software tools for evaluating reliability, availability and serviceability, *ACM Computing Surveys* **20**(4), (1988), 227–269.

[4] A. K. Aggarwal, S. Kumar, V. Singh, and T. K. Garg, Markov modeling and reliability analysis of urea synthesis system of a fertilizer plant, *Journal of Industrial Engineering International* **11**, (2015), 1-14.

[5] A. K. Aggarwal, S. Kumar, and V. Singh, Mathematical modeling and fuzzy availability analysis for serial processes in the crystallization system of a sugar plant, *Journal of Industrial Engineering International* **13**, (2017), 47-58.

[6] A. S. Maihulla, and I. Yusuf, Reliability and performance prediction of a small serial solar photovoltaic system for rural consumption using the Gumbel-Hougaard family copula, *Life Cycle Reliability and Safety Engineering* **10,** (2021), 347–354. https://doi.org/10.1007/s41872-021-00176-x

[7] A. S. Maihulla, I. Yusuf, and M. S. Isa, Reliability modeling and performance evaluation of solar photovoltaic system using Gumbel–Hougaard family copula, *International Journal of Quality & Reliability Management* **39**(8), (2021), 2041-2057. doi.org/10.1108/IJQRM-03-2021-0071

[8] A. S. Maihulla, and I. Yusuf, Reliability, availability, maintainability, and dependability analysis of photovoltaic systems, *Life Cycle Reliability and Safety Engineering* **11,** (2022), 19–26. doi.org/10.1007/s41872-021-00180-1

[9] A. S. Maihulla, and I. Yusuf, Performance analysis of photovoltaic systems using RAMD analysis, *Journal of the Nigerian Society of Physical Sciences* **3**(3), (2021), 172–180. doi.org/10.46481/jnsps.2021.194

[10] B. S. Dhillon, *Reliability Engineering in Systems Design and Operation*, Van Nostrand-Reinhold, New York, 1983.

[11] H. Sharon, M. Vivar, and M. Fuentes, A review on role of solar photovoltaic (PV) modules in enhancing sustainable water production capacity of solar distillation units, *Journal of Environmental Management* **320,** (2022), No 115781. doi.org/10.1016/j.jenvman.2022.115781.

[12] I. Yusuf, B. Sani, and B. Yusuf, Profit analysis of a serial-parallel system under partial and complete failures, *Journal of Applied Sciences* **19**, (2019), 565-574.
DOI: 10.3923/jas.2019.565.574

[13] J. A. Van, A feasibility study by means of the development of a simulation model for the resin plant of the Lexan production train of GE Plastics, MSc thesis, Delft Technical University 2003.

[14] L. S. Srinath, *Reliability Engineering*, East-West Press Pvt. Ltd, New Delhi,1994.

[15] L. C. Wolstenholme, *Reliability Modelling–A Statistical Approach*, Chapman & Hall-CRC, New York, 1999.

[16] M. Malhotra, and K. S. Trivedi, Power-hierarchy of dependability-model types, *IEEE Transactions on Reliability* **43**(3), (1994), 493–502.

[17] P. D. T. O'Conner, *Practical Reliability Engineering,* John Wiley & Sons Ltd, Chichester, 1995.

[18] M. S. Moustafa, Transient analysis of reliability with and without repair for K-out-of-N:G systems with two failure modes, *Reliability Engineering & System Safety* **53**(1), (1996), 31-35.

[19] R. Khanduja, P. C.Tewari, and D. Kumar, Availability analysis of bleaching system of paper plant, *Journal of Industrial Engineering, Udyog Pragati, N.I.T.I.E. Mumbai (India)* **32**(1), (2008).

[20] S. Gupta, P. C. Tewari, and A. K. Sharma, Performance modeling and decision support system of feed water unit of a thermal power plant, *South African Journal of Industrial Engineering* **19**(2), (2008), 125-134.

[21] S. Kumar, P. C. Tewari, and S. Kumar, Performance modelling and simulated availability of shell gasification and carbon recovery system of urea plant, *Proceedings of the 16th IASTED International Conference held in Spain*, (2007), 409-413.

[22] S. Gupta, P. C. Tewari, and A. K. Sharma, Reliability and availability analysis of ash handling system of a steam thermal tower plant, *South African Journal of Industrial Engineering* **20**(1), (2009), 147-158.

[23] S. Kumar, D. Kumar, and N. P. Mehta, Maintenance management for ammonia synthesis system in a urea fertilizer plant, *International Journal of Management and Systems* **15**(3), (1999), 211-214.

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