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Modeling & Simulation of Covid-19 Disease by Means of Chebyshev Wavelets

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Abstract. *The mathematical modeling of infectious diseases has become of paramount importance in recent years. Since 2019, with the worldwide spread of the Covid-19 pandemic, many already existing mathematical models have been improved. This paper focuses on the digital analysis of a model of transmission of the Covid-19 disease by means of Chebyshev wavelets. The analysis of the results obtained makes it possible to make a very good forecast of positive Covid-19 cases according to the parameters studied.*

Key words: Epidemiology, Covid-19 Disease, Chebyshev Wavelets, Numerical Results.

AMS Subject Classifications: 65S05, 65T60, 92B05, 92D30

1. Introduction & Notations

1.1. Introduction

Mathematical modeling and the progress of medicine, in terms of vaccinations and improvements of living conditions, in general, have suggested that infectious diseases would disappear as of the second half of the twentieth century. To everyone's surprise or dismay, other even more dangerous diseases have been discovered, or rediscovered, such as Cancer, Malaria, Yellow Fever, AIDS, Ebola, Cholera, Dengue and Chikunguny and recently the disease COVID 19. These are the names of diseases that will forever stay in the memory of mankind. Infectious diseases, as directly or indirectly transmissible by viruses and bacteria, are one of the areas where the theoretical foundations have been most developed in epidemiology. These diseases call for an understanding of their dynamics before proceeding to control of their propagation in populations.

The mathematical modelling of infectious diseases remains a relatively new science. While epidemiology has a long history, it is only recently that mathematicians, epidemiologists, and immunologists have begun to collaborate to create models that can predict the course of a disease. The first model was developed by Bernoulli in 1760 for smallpox in [1].

The principles of the mathematical epidemiology approach based on compartmental models have been established by public health physicians such as Ronald Ross, W. H. Hamer, [2], and W.O. Kermack, [3]. Ronald Ross, [4], can be considered the founding father of current modelling in epidemiology. He was awarded the Nobel Prize in 1902 for his proof that malaria was transmitted by Anopheles.

The use of mathematical models remains quite old in epidemiology, but in recent years, there has been a very significant increase in the number of publications using these models in the analysis of the spread and control of infectious diseases. In fact, a model is a formulation of mathematical equations of a phenomenon arising from the real world. A good model is therefore the translation of reality using differential equations or systems of such equations.
Recently a particular interest is brought to wavelets and their applications in numerical analysis. This prompted us to use Chebyshev's wavelets to study the epidemiological evolution of the COVID 19 disease. Epidemiological mathematical models are most often governed by systems of partial differential equations but before that, it would be good, or necessary, to define the types of models in epidemiology that may enable characterization of COVID-19 according to its causes, its manifestations and modes of transmission. In [5], Kumar and al. propose a non-signular fractional derivative and a Legendre colocation methode to solve a given "Covid-19" model.

The rest of the paprer is organized as follows: Section 2 deals with mathematical modeling of the Corona-virus disease. In section 3, we review Chebyshev polynomials and wavelets and their numerical analysis. Section 4 is devoted to mathematical analysis of the Covid-19 model. And section 5 deals with numerical analysis of the model with Chebyshev wavelets and some numerical simulations. Section 6 contains some coments and concluding remarks.

1.2. **Notations**

Throughout this work the following notations are used:
$S(t)$: *Susceptible,* individuals receptive to the infectious agent who are not contaminated but may become so;
$I(t)$: *Infected,* individuals affected and who are therefore contagious;
$R(t)$: *Recovered;*
$Q(t)$: *Quarantined;*
$D(t)$: *Death;*
β : *Infectious contact rate*
γ : *Cure rate;*
α : *Rate of change from infectious to infected.*

2. Modeling of the Corona Virus Disease (Covid-19)

2.1. **The corona virus disease**

In December 2019, an outbreak of "unknown" viral pneumonia was detected in the city of Wuhan, the sixth largest city in China with 19 million inhabitants (Hubei province). The first patients were hospitalized on December 16th, however Chinese authorities did not notify the World Health Organization until December 31st, 2019. Between December 12th and 29th, 2019, nearly 59 cases of pneumonia were finally reported. Seven patients remained in critical condition, no deaths were reported: however 163 people in contact with the sick had been placed under observation. At first, Chinese authorities suspected the resurgence of Severe Acute Respiratory Syndrome (SARS). This syndrome had already caused a pandemic in Asia in 2002-2003, infecting more than 8,000 people and causing the death of 774 people in 37 countries. However, on January 5, 2020, the Chinese government ruled out this hypothesis. SARS was out of the question, which raised the question of the origin of the epidemic. Research will then turn to the seafood market, which also offered many fresh animal chairs (fox, rat, snake, camel meat, etc.). If contamination of animal origin is thus privileged (SARS began in 2002 with the contamination of a human by a wild animal, the Civet Cat or ˜civette" in French, a dish very appreciated in the southern regions of China), one question remained unanswered: the possible human-to-human transmission, even if the closure of the incriminated market suggested that the risk was limited.

On January 9th, the discovery of a new corona virus (first called 2019-n CoV), then officially SARS-Cov2, was announced by Chinese health authorities and the WHO. This new virus is the causative agent of the new infectious respiratory disease called "COVID-19" (Corona Virus Disease). Although there is still insufficient evidence of human-to-human transmission, fears of an epidemic nationwide and in neighbouring countries are present. The days following the announcement of this discovery will only heighten concerns, especially with the announcement of the first two deaths in the city of Wuhan by the Chinese authorities (January 19th, 2020). Neighbouring countries are not spared. More than 90 suspected cases have been reported in Hong Kong and four in Taiwan (International Courier 8 of January 18, 2020). On the same day three travellers, two from Thailand and one from Japan who visited Wuhan but who were not in the market, were infected with the virus. Human-to-human transmission is confirmed. Researchers at the Center for Infectious Disease Analysis at Imperial College London, in the person of Professor Neil Ferguson, do not hesitate to consider that the number of infected patients may well be higher than the few dozen reported cases.

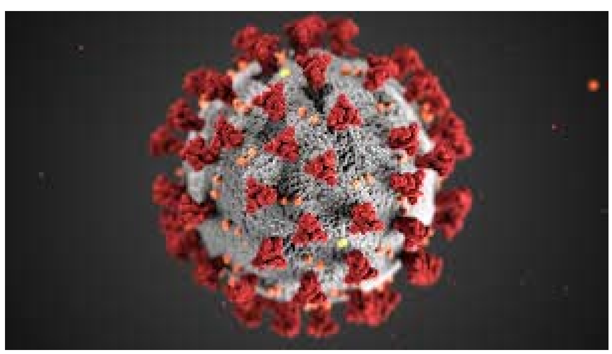


Figure 1 : Sketch for a Covid-19 image.

Many countries have started to take preventive measures. The Hong Kong authorities have thus put in place screening measures, in particular strict temperature controls for travelers from Mainland China, a protocol already in place during SARS 2003 (Bangkok Post 11 of January 18th, 2020). Thailand said it was already screening passengers disembarking in Bangkok, Chiang Mai and Phuket (Bangkok Post 12). After a first case announced on their territory, on January 21st, the United States also set up a control system (questionnaire on possible respiratory symptoms and temperature measurement) for all passengers coming from Wuhan at airports. of New York, San Francisco and Los Angeles (Los Angeles Times 13, January 18th, 2020).

On January 22, 2020, WHO met urgently to decide whether the 2019 coronavirus pneumonia outbreak - NcoV constitutes a public health emergency of international concern. At the same time, Chinese authorities are announcing the decision to quarantine the Wuhan region, the epicenter of the epidemic.

This period is also marked by scientific advances. The study of numerous genomic sequences of 2019-nCoV makes it possible to trace with a greater probability its genealogy which brings it closer to Coronivarus HK9-1 (discovered in China in 2011), a virus detected in a bat 16 (Le Monde 17 of January 25th). The animal responsible for transmission to humans has not yet been identified with certainty, however some publications (Xiao, [6], 2020; Lam, [7], 2020) suggest that the small mammal pangolin consumed in southern China for its tender flesh could be involved as an "intermediate host" between the bat and man (Institut Pasteur 18). As for symptoms, they were also the subject of an initial census (Institut Pasteur, Ibid.):

- incubation duration is on average 5-6 days (with extreme values ranging from 2 to 14 days), symptoms gradually set in;
- the first signs are not very specific (they are like the flu): headache, muscle pain, fatigue. Fever and respiratory signs then occur (two - three days later);
- early studies from China found that an average of one week elapsed between the onset of symptoms and admission to the status stage. At this stage, the symptoms combine fever, cough, chest pain and difficulty breathing (performing a chest CT scan almost always reveals pneumonia affecting both lungs;
- The seriousness of the clinical signs requires that approximately 20/100 patients be kept in hospital and 5/100 require admission to intensive care;
- Some studies and models seem to show that the infection can be asymptomatic (not leading to clinical manifestations) in 30/100 to 60/100 of infected subjects.

Beyond these scientific advances, the virus continued to spread beyond the borders of the Wuhan market. As of January 27th, it had infected nearly 4,400 people, including around 40 overseas in 18 different countries (resulting in 107 deaths). France confirms the presence of three cases, two Chinese tourists from Wuhan and a Frenchman of Chinese origin who also passed through the same city (Le Monde 19, January 25th, 2020). The month of February confirms the epidemic outbreak with outbreaks (The John Hopkins CSSE counted 82,446 cases of contamination and 2,808 deaths on February 27th, 2020) which intensified in China (78,497 cases of contamination), South Korea (1,766 cases), Japan (189 cases), Singapore (93 cases) and Hong Kong (92), and outbreaks in Iran (245 cases) and Italy (453 cases). The Italian case is the subject of special attention (on February 19th, 2020, Italy had only 3 infected people).

This epidemic very quickly became a pandemic and in a few months, there were few countries which did not have positive cases of Covid-19. However, there is not yet an effective, accurate and WHO-recognized treatment, although some countries such as Senegal prefer to use hydroxy-chloroquine. Just as the Malagasy remedy called artemisia is also in great demand here in Africa for the treatment and even as a preventive vaccine for Covid-19.

To date (16 August 2021), the world has 207 million of confirmed cases including 4.37 million dead from Covid-19 and the United States becomes the most affected country with more than 36.7 million of confirmed cases and a sad toll of 621,000 dead. At the same time, in Senegal there are 70,854 (to date) cases declared positive of which 54,357 cured, 1,614 died and 14,882 in treatment.

2.2. **Mathematical modeling of Covid-19**

In general, epidemiological modeling is based on compartmental aspects. The population is divided into different compartments according to the behavior in relation to the disease. To characterize the "COVID-19" pandemic, we will use seven compartments: S, I, E, D, R, Q and possibly P that we will define in a particular way as we move forward in the description of the model. Compartment S is necessary, since it must initially exist in the population of individuals who have not yet been infected. When an S-compartment individual is exposed to the disease, he or she does not necessarily become able to transmit it immediately, depending on the timescale considered in the model. For example, if covid-19 disease takes 14 days to make the individual infectious (this is called a two week latency period) and the model represents the daily change in the population, then the individual does not go directly to compartment I (infectious) but must pass through an intermediate compartment. Such a compartment is denoted E, for exposed individuals. Then, among infected individuals, it is assumed that if the disease is caused by parasitic organisms such as parasitic worms or ticks, then the concentration of these organisms can justify dividing class I into several classes representing several levels of concentration. When the disease is caused by viruses or bacteria, a large number of models do not divide class I: they consider the individual to be infected or not. However, in the case of a virus and, moreover, that of Covid-19, there is an analogue to the concentration of parasitic organisms: this is the viral load, which expresses the concentration of the virus in a given volume such as the blood. Individuals can therefore be distinguished according to their load, since this makes them infectious at different levels, because research has shown that some people can be infected and without treatment and they are able to recover from the disease without the need for treatment. However, after an individual has been infected and detected, he will be automatically quarantined in order to be able to treat him but above all to avoid further contamination, which means that the individual can still be infectious but he will find himself isolated from the population by quarantine policy, hence the need to provide another compartment Q and in this compartment Q, two cases can occur: first, the individual can die, in which case he falls under compartment D. Second, the disease may terminate on its own and confer immunization against re-infection in the individual, and it is assigned to an R compartment.

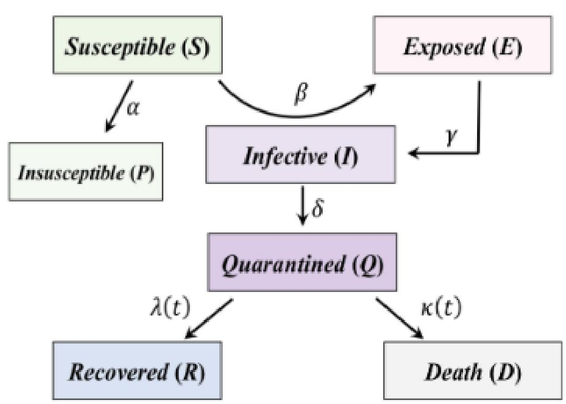


Figure 2 : Proposed epidemical scheme for Covid-19.

Separating the dead individuals from the recovered individuals from the R compartment also makes it possible to take into account the fact that the latter may possibly re-enter the S compartment. And finally, to take into account the improvement of public health and possibly another P compartment in which we assume that part of the population is fully protected due to compliance with barrier measures, such as promoting the wearing of face masks, more effective contact tracing and stricter lockdowns of communities, we assume that the sensitive population decreases stably and therefore introduce a positive protection rate α in the model.

The relationships defined in Figure 2 are characterized by a group of the following ordinary differential equations:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta\dfrac{S(t)I(t)}{N} - \alpha S(t) \\ \dfrac{dE(t)}{dt} = \beta\dfrac{S(t)I(t)}{N} - \gamma E(t) \\ \dfrac{dI(t)}{dt} = \gamma E(t) - \delta I(t) \\ \dfrac{dQ(t)}{dt} = \delta I(t) - \lambda(t)Q(t) - \kappa(t)Q(t) \\ \dfrac{dR(t)}{dt} = \lambda(t)Q(t) \\ \dfrac{dD(t)}{dt} = \kappa(t)Q(t) \\ \dfrac{dP(t)}{dt} = \alpha S(t) \end{cases} \tag{1}$$

The constant $N = S + P + E + I + Q + R + D$ is the total population in a certain area. The coefficients α, β, γ, δ, λ and κ separately represent the protection rate, infection rate, mean latent time, mean quarantine time, cure rate and mortality rate.

In the following section, we introduce the Chebyshev wavelets, that we will use for numerical analysis of the system (1).

3. Chebyshev Wavelets

Wavelets are special functions in a limited domain that is, a wave function instead of oscillating forever it drops to zero. Recently, we have facing different kinds of wavelets which are depending on two parameters such as, n is dilation parameter and k is the translation parameter. The theory and application of wavelets is a comparatively young branch in signal processing and mathematical field. It has been applied in engineering disciplines, such as signal analysis, time-frequency analysis, and engineering mathematics. For example, Kumbinarasaiah et al., in [8], used a Laguerre wavelet to propose a numerical method for solving an SIER model for Covid-19. In [9], Wang et al. used Haar wavelets to solve fractional partial differential equations. In this paper, we use Chebyshev wavelets to solve the system of equations that model Covid-19.

Definition 3.1. For n and $m = 0, 1, \ldots, M-1$ and $n = 1, 2, \ldots, 2^{2k-1}$, Chebyshev polynomials of the first kind are polynimials T_m that satisfy the following recurrence formula :

$$\begin{cases} T_0(t) = 1 \\ T_1(t) = t \\ T_{m+1}(t) = 2tT_m(t) - T_{m-1}(t), \quad m = 1,2,3,\ldots \end{cases} \tag{2}$$

Proposition 3.1. *Chebyshev polynomials are orthogonal over the interval* $]-1;1[$ *with the weighting function* $\frac{1}{\sqrt{1-x^2}}$. *More precisely, they satisfy*

$$\int_{-1}^{1} T_m(x)T_n(x)dx = \begin{cases} 0 & si \quad m \neq n \\ \pi & m = n = 0 \\ \frac{\pi}{2} & m \neq n \neq 0 \end{cases} \tag{3}$$

and are solutions to the following differential equation:

$$(1-x)T_n''(x) - xT_n'(x) + n^2 T_n(x) = 0.$$

Wavelets constitute a family of functions constructed from dilation and translation of a single function called the mother wavelet, [10-12]. When the dilation parameter, a and the translation parameter, b, vary continuously, we have the following family of continuous wavelets as

$$\psi_{a,b}(t) = |a|^{\frac{1}{2}}\psi\left(\frac{t-b}{a}\right), \quad a,\ b \in \mathbb{R}, \quad a \neq 0. \tag{4}$$

If we take the dilation and translation parameters a^k, and nba^k, respectively where $a > 1,\ b > 0,\ n$, and k are positive integers, then we have the following family of discrete wavelets

$$\psi_{k,n}(t) = |a|^{\frac{1}{2}}\psi(a^k t - nb). \tag{5}$$

Definition 3. 2. Chebyshev wavelets $\Psi_{n,m}(t) = \Psi(k,n,m,t)$ are defined by the following formula:

$$\Psi_{n,m}(t) = \begin{cases} 2^{\frac{k}{2}}\widetilde{T}_m\left(2^k t - 2n + 1\right) & \frac{n-1}{2^{k-1}} \leq t \leq \frac{n}{2^{k-1}} \\ 0, & otherwise, \end{cases} \tag{6}$$

where

$$\widetilde{T}_m(t) = \begin{cases} \frac{1}{\sqrt{\pi}} & if \quad m = 0 \\ \sqrt{\frac{2}{\pi}}\, T_m(t), & if \quad m > 0, \end{cases} \tag{7}$$

with $n = 1,\ldots 2^k$ represents the number of levels of the decomposition, $k \in \mathbb{N}^*$, t is the temporal variable. T_m is the Chebyshev wavelet of degree $m = 0,\ldots,M-1$ (where $M \in \mathbb{N}^*$ is the number of collocation points).

Remark 3. 1.

1. The coefficients in equation (7) ensure the orthonormality.

2. In order to obtain an orthogonal Chebyshev wavelet family in Hilbert space $L^2_{w_n}([0;1])$ the weighting function must be translated and expanded as follows:

$$w_n(t) = w(2^k t - 2n + 1)$$

The set $\{\Psi_{nm}(t)\}_{n,m\in \mathbb{N}}$ forms a basis in Hilbert space $L^2_w([0,1])$, where the $L^2_w([0,1])$ space is defined by:

$$L^2_w([0,1]) = \left\{ f \in L^2([0,1]);\ \int_0^1 |f(x)|^2 w(x)dx < \infty \right\}$$

equipped with the standard

$$\| f \|_{2,w} = \left(|f(x)|^2 w(x)dx \right)^{\frac{1}{2}},$$

with the associated scalar product :

$$< f,g >_{2,w} \int_0^1 f(x)g(x)w(x)dx.$$

3.1. **Approximation by the orthogonal function**

Particular attention has been paid to applications of orthogonal functions such as Legendre polynomials, Laguerre polynomials and Chebyshev polynomials. There are three classes of orthogonal function sets that are widely used:

– The first includes sets of piecewise constant base functions (such as Walsh functions, block pulse functions, etc.).

– The second concerns the sets of orthogonal polynomials (such as Legendre polynomials and Chebyshev polynomials etc...

– The third comprises the widely used sets of Fourier serial sine and cosine functions.

The main feature of using orthogonal functions is to reduce the problems to be solved to a system of linear algebraic equations with a sparse matrix.

3.2. **Approximation of a function of function** $L^2([0,1])$

Since the wavelet family (6) forms a basis of $Ł^2([0,1]))$ then any function f of this space can be written as follows::

$$f(t) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} C_{n,m} \psi_{n,m}(t), \tag{8}$$

where C et Ψ are matrices of dimension $2^{k-1} \times 1$ given by:

$$C = \left[C_{1,0}, C_{1,1}, \dots, C_{1,M-1}, C_{2,0}, \dots, C_{2^k 0}, \dots, C_{2^j-1,0}, \dots, C_{2^k-1,M-1} \right]^T , \tag{9}$$

$$\psi(t) = \left[\psi_{1,0}, \psi_{1,1}, \dots, \psi_{1,M-1}, \psi_{2,0}, \dots, \psi_{2M-1}, \dots, \psi_{2^j-1,0}, \dots, \psi_{2^k,0}, \dots, \psi_{2^k-1,M-1} \right]^T. \tag{10}$$

4. Mathematical Analysis of the Covid-19 Model

The mathematical model of Covid-19 is described by the following differentiel system:

$$\begin{cases} \dot{S}(t) = -\beta \dfrac{S(t)\, I(t)}{N} - \alpha S(t) \\ \dot{E}(t) = \beta \dfrac{S(t)\, I(t)}{N} - \gamma E(t) \\ \dot{I}(t) = \gamma E(t) - \delta I(t) \\ \dot{Q}(t) = \delta I(t) - \lambda(t) Q(t) - k(t) Q(t) \\ \dot{R}(t) = \lambda(t) Q(t) \\ \dot{D}(t) = \kappa(t) Q(t) \\ \dot{P}(t) = \alpha(t) S(t). \end{cases} \tag{11}$$

It is clear from (11) that the total population N is constant. We eliminate R by using $R = N - I - S - D - P - E$ and $\overline{\beta} = \frac{\beta}{N}$ to obtain the equivalent system

$$\begin{cases} \dot{S}(t) = -\overline{\beta} S(t) I(t) - \alpha S(t) \\ \dot{E}(t) = \overline{\beta} S(t) I(t) - \gamma E(t) \\ \dot{I}(t) = \gamma E(t) - \delta I(t) \\ \dot{Q}(t) = \delta I(t) - \lambda(t) Q(t) - k(t) Q(t) \\ \dot{D}(t) = \kappa(t) Q(t) \\ \dot{P}(t) = \alpha(t) S(t). \end{cases} \tag{12}$$

4.1. **Existence**, **uniqueness**, **positivity and boundedness of the solution**

Here, we study the basic results of solutions of model system (12), which are essential for the well posedness of the system and shall be useful in the investigation of the long-run dynamics. We have the following result.

Theorem 4. 1. *The non-negative orthant R^6_+ is positively invariant for model system (12). More precisely, non-negative solutions of model system stay non-negative for all the time.*

Proof. We show that the solution $S(t)$, of model system (12) corresponding to the initial condition $S(0) > 0, E(0) > 0, I(0) > 0, Q(0) > 0, D(0) > 0$ and $P(0) > 0$ is non-negative. Let $\tau = \sup\{t > 0 : \ S(t) > 0, E(t) > 0, I(t) > 0, Q(t) > 0, D(t) > 0, P(t) > 0\}$. If $\tau = \infty$, then all solutions of the system (12) are positive.
Suppose $\tau < \infty$, there is at least one solution $S(t)$ or which is equal to zero(from the definition of τ as a supremum).
Suppose $E(\tau) = 0$ and let us consider the following equation of model system (12):

$$\dot{E}(t) = \overline{\beta} S(t) I(t) - \gamma E(t).$$

We know that for all $t \in [0,\tau]$, $\overline{\beta} S(t) I(t) \geq 0$. It follows that

$$\dot{E}(t) + \gamma E(t) \geq 0.$$

Therefore,

$$\begin{aligned}&\frac{d}{dt}[E(t)exp(\gamma t)]\\&= \dot{E}(t)exp(\gamma t) + \gamma E(t)exp(\gamma t)\\&= exp(\gamma t)(\dot{E}(t) + \gamma E(t)) \geq 0.\end{aligned}$$

$$\text{i.e. } \frac{d}{dt}[E(t)exp(\gamma t)] \geq 0.$$

Integrating the above inequality from 0 to τ gives

$$\int_0^\tau \frac{d}{dt}[E(t)exp(\gamma t)] \geq 0,$$

or equivalently

$$E(\tau)\ exp(\gamma\tau) - E(0) \geq 0.$$

$$E(\tau) \geq exp(\gamma\tau)\ E(0) > 0,$$

which is in contradiction with $E(\tau) = 0$. The others cases lead to the same contradiction. Thus $S(t) > 0, E(t) > 0, I(t) > 0, Q(t) > 0, D(t) > 0$ and $P(t) > 0$ for all $t > 0$. This completes the proof. ■

We can now show that model system (12) is mathematically and epidemiologically well posed in the following theorem. Let $Z(t) = S(t) + E(t) + I(t) + Q(t) + D(t) + P(t) \leq N$.
To simplify the analysis of the model system (12), we work with fractional quantities instead of actual populations by scaling the population of each class by the total species population N. Note that $s(t) = \frac{S(t)}{N}, i(t) = \frac{I(t)}{N}, e(t) = \frac{E(t)}{N}, q(t) = \frac{Q(t)}{N}, d(t) = \frac{D(t)}{N}$ and $p(t) = \frac{P(t)}{N}$. and consider the domain

$$\mathsf{D} = \{(s,e,i,q,d,p) \in \mathsf{R}_+^6 | z(t) = s(t) + e(t) + i(t) + q(t) + d(t) + p(t) \leq 1\},$$

to state the next result.

Theorem 4. 2. *Assuming that the initial conditions lie in D, the system of equations for the model (12) has a unique solution that exists and remains in D for all time $t \geq 0$.*

Proof. First, note that the right-hand side of model system (12) is locally Lipschitz. Thus, by the classical results of dynamical systems, for any initial condition $(S(0), E(0), I(0), Q(0), D(0), P(0) > 0)$, the associated Cauchy problem with model system (12) has a unique. This solution is always positive by the theorem (12). We now show that D is forward-invariant. We can see from (12) that if $s = 0$ then $\dot{s} = 0$; if $e = 0$ then $\dot{s} \geq 0$; $i = 0$ then $\dot{i} \geq 0$ and $q = 0$ then $\dot{q} \geq 0$. It Also true that if $s(t) + e(t) + i(t) + q(t) + d(t) + p(t) = 1$ then $\frac{d}{dt}(s(t) + e(t) + i(t) + q(t) + d(t) + p(t)) < 0$ because $-\lambda(t)q(t) < 0$. for all $t > 0$. Therefore, none of the orbits can leave D, and a unique solution exists for all time. ■

4.2. **Global stability of solutions**

To prove the stability of the model (1), we will consider the Lyapunov function constructed as follows:

$$L = B_1 E + B_2 I$$

$B_1 = \gamma\delta$ and $B_2 = \frac{\beta}{N}\gamma$

$$\frac{\partial L}{\partial t} = B_1\left[\beta\frac{S(t)\,I(t)}{N} - \gamma E(t)\right] + B_2[\gamma E(t) - \delta I(t)]$$

$$= \gamma\delta\left[\beta\frac{S(t)\,I(t)}{N} - \gamma E(t)\right] + \frac{\beta}{N}\gamma[\gamma E(t) - \delta I(t)]$$

$$= \gamma\delta\frac{\beta}{N}S(t)I(t) - \gamma^2\delta E(t) + \frac{\beta}{N}\gamma^2 E(t) - \frac{\beta}{N}\gamma\delta I(t)$$

$$\leq -\gamma^2\delta E(t) + \frac{\beta}{N}\gamma^2 E(t) - \frac{\beta}{N}\gamma\delta I(t)$$

$$\leq -\gamma^2\delta E(t) - \frac{\beta}{N}\gamma\delta I(t) < 0$$

$$\Rightarrow \frac{\partial L}{\partial t} \leq 0$$

As $\frac{\partial L}{\partial t} \leq 0$, then by the LaSalle invariance principle, we can conclude that our solution of model (1) is globally and asymptotically stable.

5. Numerical Analysis of the Covid-19 Model by Chebyshev Wavelets

5.1. **Operational integration matrix**

The basic ideas of the operational integration matrix are

1. The differential equation is converted to an integral equation through multiple integration;
2. Subsequently, the various functions concerned by the integral equation are approximated by a representation in the form of linear combinations of orthonormal basis functions and truncating them to an optimal level;
3. Finally, the integral equation is converted into a system of algebraic equations by the introduction of the operational matrix for the integration of basic functions.

The integral of Ψ between 0 and t can be expressed as a function of Ψ and through the operational integration matrix P.

$$\int_0^1 \Psi(s)ds = P\,\Psi(t). \tag{13}$$

The matrix P is represented as

$$P = \begin{pmatrix} L & F & F & F & \ldots\ldots & F \\ 0 & L & F & F & \ldots & F \\ 0 & 0 & L & F & \ldots & F \\ 0 & 0 & 0 & L & F\ldots & F \\ . & . & . & . & \ldots. & . \\ . & . & . & . & \ldots & . \\ . & . & . & . & \ldots & . \\ . & . & . & . & \ldots & . \\ 0 & 0 & 0 & 0 & \ldots & L \end{pmatrix}$$

with

$$L = \begin{pmatrix} 1 & \frac{1}{\sqrt{2}} & 0 & 0 & \ldots & 0 & \\ -\frac{\sqrt{2}}{4} & \frac{1}{4} & 0 & 0 & \ldots & 0 & \\ -\frac{\sqrt{2}}{3} & -\frac{1}{2} & 0 & \frac{1}{6} & \ldots & 0 & \\ . & . & . & . & \ldots & . & \\ . & . & . & . & \ldots & . & \\ . & . & . & . & \ldots & . & \\ . & . & . & . & \ldots & . & \\ \frac{\sqrt{2}}{2}\left(\frac{(-1)^{r-1}}{r-1} - \frac{(-1)^{r+1}}{r+1}\right) & 0 & 0 & \ldots\frac{1}{2(r-1)} & 0 & \frac{1}{2(r+1)} & \\ . & . & . & . & \ldots & . & \\ \frac{\sqrt{2}}{2}\left(\frac{(-1)^{r-2}}{r-2} - \frac{(-1)^{M}}{M}\right) & 0 & 0 & 0 & \ldots & -\frac{1}{2(M-2)} & 0 \end{pmatrix}$$

and

$$F = 2^{-k}\begin{pmatrix} 2 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ -\frac{2\sqrt{2}}{3} & 0 & 0 & 0 & \dots & 0 \\ . & . & . & 0 & 0\dots & 0 \\ . & . & . & . & \dots & . \\ . & . & . & . & \dots & . \\ . & . & . & . & \dots & . \\ \frac{\sqrt{2}}{2}\left(\frac{1-(-1)^{r+1}}{r+1} - \frac{1-(-1)^{r-1}}{r-1}\right) & 0 & 0 & \dots\frac{1}{2(r-1)} & 0 & 0 \\ . & . & . & . & \dots & . \\ . & . & . & . & \dots & . \\ . & . & . & . & \dots & . \\ \frac{\sqrt{2}}{2}\left(\frac{1-(-1)^{M}}{M} - \frac{1-(-1)^{M-2}}{M-2}\right) & 0 & 0 & \dots & 0 & 0 \end{pmatrix}$$

with L and F are two square matrices of dimension $(M \times M)$.
To determine the integration matrix P, we integrate each component $\psi_{n,m}$ of the vector (15) on $[0,t]$ and we express the result of this integration as a function of the components of this vector.
We can also verify that this matrix is invertible. with $C_{n,k} = \langle f, \psi_{n,k}\rangle$ such that $\langle .,.\rangle$ denotes the scalar product in $L^2([0,1])$.
We truncate the series (8), we obtain:

$$f(x) \approx \sum_{n=1}^{2^{j-1}} \sum_{k=0}^{M-1} C_{n,k}\psi_{n,k}(x) = C^T\psi(x),$$

where C and ψ are vectors of dimension $2^{k-1}M \times 1$ given by:

$$C = \left[C_{1,0}, C_{1,1}, \dots, C_{1,n-1}, C_{2,0}, \dots, C_{2,M-1}, \dots, C_{2^{k-1},0}, \dots, C_{2^{j-1},M-1}\right]^T, \tag{14}$$

$$\psi(x) = \left[\psi_{1,0}, \psi_{1,1}, \dots, \psi_{1,M-1}, \psi_{2,0}, \dots, \psi_{2,M-1}, \dots, \psi_{2^{k-1},0}, \dots, \psi_{2^{k-1},M-1}\right]^T. \tag{15}$$

Each equation of the system (1) can be written in the form:

$$a(t)U'(t) + b(t)U(t) = f(t) \qquad \text{for } t \in]0;1]. \tag{16}$$

For example for S, we have :

$$S' = -\beta\frac{I}{N}S - \alpha S$$

$$S' + \left(\beta\frac{I}{N} + \alpha\right)S = 0$$

$$a(x) = 1 \quad b(x) = (\lambda(t) + \kappa(t)) \quad f(x) = \delta I(t)$$

with the initial condition

$$U(0) = U_0,$$

where a, b and f are continuous functions. To solve the problem, we use the wavelet decomposition, we set

$$U'^T\Psi(t)$$

and

$$\begin{cases} a(t) = A^T\Psi(t) \\ b(t) = B^T\Psi(t) \\ f(t) = F^T\Psi(t). \end{cases} \quad (17)$$

By integrating the two sides of $(0,t)$, we find :

$$\int_0^t U'(s)ds = \int_0^t U^T\Psi(s)ds$$

$$U(t) - U(0) = U^TP\Psi(t)$$

$$U(t) = U^TP\Psi(t) + U(0)$$

and

$$U(t) = \left(U^TP + U_0d^T\right)\Psi(t), \quad (18)$$

where $1 = d^T\Psi(t)$.

We recall the property of the product of two Chebyshev wavelets [13] :

$$A^T\Psi^{(i)}(t)\left(\Psi^{(i)}\right)^T(t) = \left(\Psi^{(i)}\right)^T(t)\overline{A}, \quad (19)$$

where A is a vector and $\tilde{A}$ is a matrix of dimension $(2^{j-1}M) \times (2^{j-1}M)$ which depends on the vector A, [10]. Substituting this expression in equation (16), we have:

$$A^T\Psi(t)\Psi^T(t)U + B^T\Psi(t)\Psi^T(t)(U^TP + U_0d^T)^T = F^T\Psi(t)$$

From (16) we have:

$$\Psi^T(t)\overline{A}U + \Psi^T(t)\overline{B}(P^TU + dU_0) = \Psi^T(t)F$$

And finally, we get a linear algebraic system::

$$(\overline{A} + \overline{B}P^T)U = (F - U_0\overline{B}d). \quad (20)$$

The solution of the problem is given by substituting the solution of the system (16) at 0. This to solve a differential equation, to solve a system then: The technical solution of decoupling and quasi-linearization consists in giving an initial profile for solution, $S^0(t)$, $E^0(t)$, $I^0(t)$, $Q^0(t)$, $R^0(t)$, $D^0(t)$, and $P^0(t)$.

5. 2. **Algorithm**

Initial data: $k = 0$

$$S^0(t),\ E^0(t),\ I^0(t),\ Q^0(t),\ R^0(t),\ D^0(t)$$

The iterative process can be written as

$$\begin{cases} \dfrac{dS^{(k+1)}}{dt} + \left(\beta \dfrac{I^k}{N} + \alpha\right) S^{(k+1)} = 0 & \dfrac{dS^{(1)}}{dt} + \left(\beta \dfrac{I^0}{N} + \alpha\right) S^{(1)} = 0 \\ \dfrac{dE^{(k+1)}}{dt} + \gamma E^{(k+1)} = \beta \dfrac{I^k}{N} S^{(k)} & \dfrac{dE^{(1)}}{dt} + \gamma E^{(1)} = \beta \dfrac{I^0}{N} S^{(0)} \\ \dfrac{dI^{(k+1)}}{dt} + \delta I^{(k+1)} = \gamma E^{(k)} & \dfrac{dI^{(1)}}{dt} + \delta I^{(1)} = \gamma E^{(0)} \\ \dfrac{dQ^{(k+1)}}{dt} + (\lambda + \kappa) Q^{(k+1)} = \gamma I^{(k)} \quad ,\dots,\dots, & \dfrac{dQ^{(1)}}{dt} + (\lambda + \kappa) Q^{(1)} = \gamma I^{(0)} \\ \dfrac{dR^{(k+1)}}{dt} = \lambda Q^{(k)} & \dfrac{dR^{(1)}}{dt} = \lambda Q^{(0)} \\ \dfrac{dD^{(k+1)}}{dt} = \kappa Q^{(k)} & \dfrac{dD^{(1)}}{dt} = \kappa Q^{(0)} \\ \dfrac{dP^{(k+1)}}{dt} = \alpha S^{(k)} & \dfrac{dP^{(1)}}{dt} = \alpha S^{(0)} \end{cases} \tag{21}$$

where $S^{(k)}$, $S^{(k+1)}$ are approximations of the solution S, at the current and previous iteration. All of these equations are decoupled in the form:

$$a(t)\frac{du(t)}{dt} + b(t)u(t) = f(t)$$

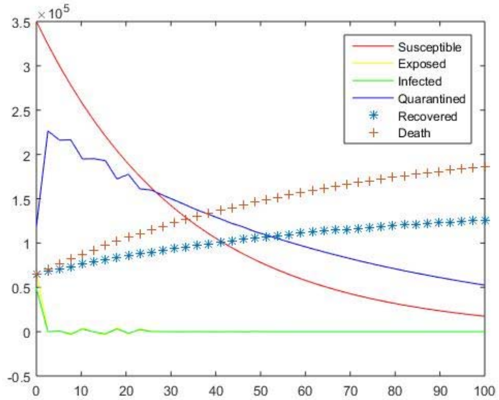
with $a(t)$, $b(t)$ and $f(t)$ known, we can use Chebyshev's wavelet method to solve it. Each equation is solved independently of the others. So, we can calculate the decoupling

$$Error_1 = Max(\| S^{(1)} - S^{(0)} \|_2 , \| E^{(1)} - E^{(0)} \|_2 , \| I^{(1)} - I^{(0)} \|_2 , \dots , \| P^{(1)} - P^{(0)} \|_2)$$

If $Error_1 < \epsilon$, then, stop and the solution is $(S^{(1)}, E^{(1)}, I^{(1)}, Q^{(1)}, R^{(1)}, D^{(1)}, P^{(1)},)$.
Else, set $k \leftarrow k + 1$.

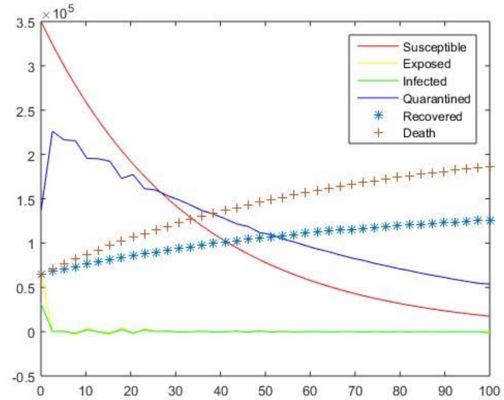
5. 3. **Numerical results**

Results of computations by using the the algorithm of (5.2) are summarized graphically below. Each figure exhibits a plot of the behavior of the population in different compartments, corresponding to a particular set $\{\alpha, \beta, \gamma, \delta, \lambda, n\}$ of parameters that characterize a specific test.



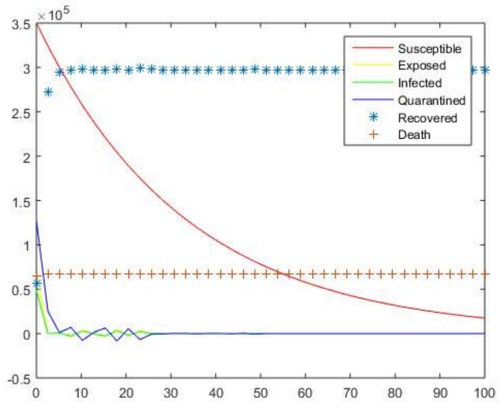
Param.	α	β	γ	δ	λ	n
Value	0.03	0.5	6	14	0.005	0.01

Figure 3 :Test 1.



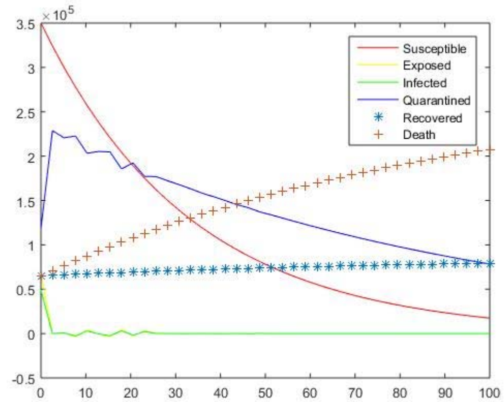
Param.	α	β	γ	δ	λ	n
Value	0.03	0.5	6	25	0.005	0.01

Figure 4 :Test 2.



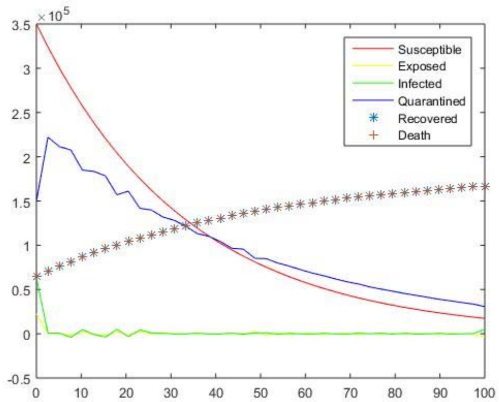
Param.	α	β	γ	δ	λ	n
Value	0.03	0.0	6	14	0.9	0.01

Figure 5 :Test 3.



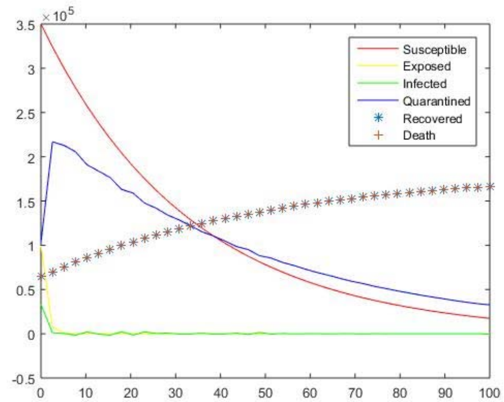
Param.	α	β	γ	δ	λ	n
Value	0.03	0.9	6	14	0.001	0.01

Figure 6 :Test 6.



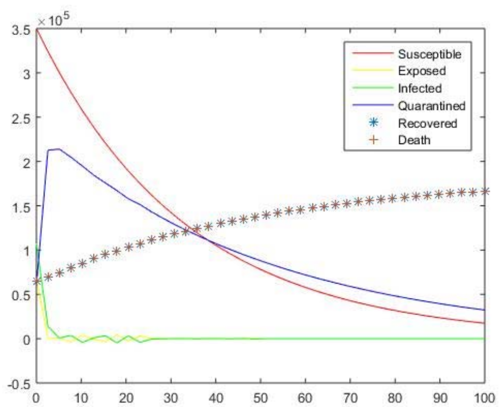
Param.	α	β	γ	δ	λ	n
Value	0.03	0.9	30	14	0.01	0.01

Figure 7 :Test 7.



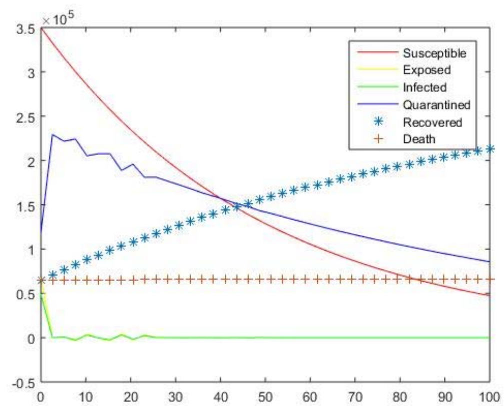
Param.	α	β	γ	δ	λ	n
Value	0.03	0.9	1	14	0.03	0.01

Figure 8 :Test 8.



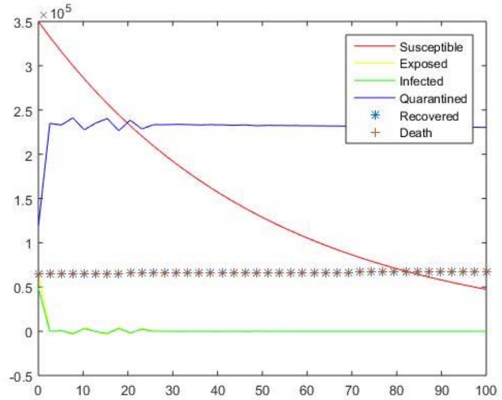
Param.	α	β	γ	δ	λ	n
Value	0.03	0.9	9	1	0.01	0.01

Figure 9 :Test 9.



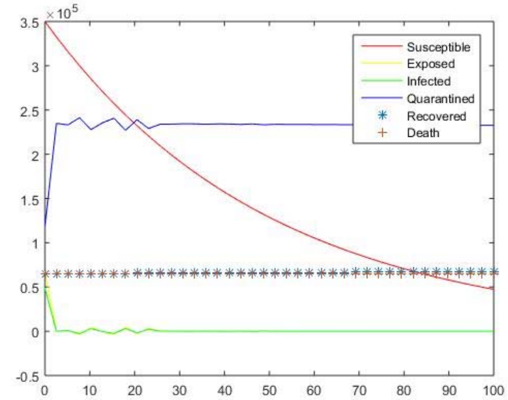
Param.	α	β	γ	δ	λ	n
Value	0.02	10^{-4}	6	14	0.01	10^{-4}

Figure 10 : Test 10.



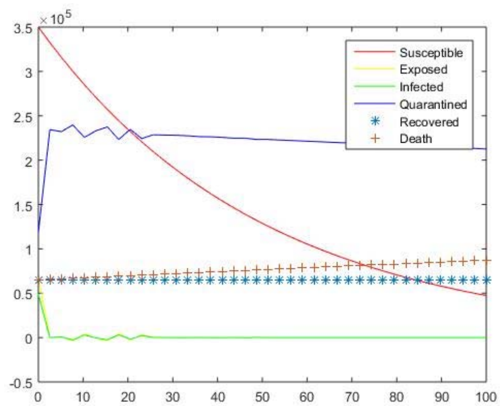
Param.	α	β	γ	δ	λ	n
Value	0.02	0.001	6	14	10^{-4}	10^{-4}

Figure 11 :Test 12.



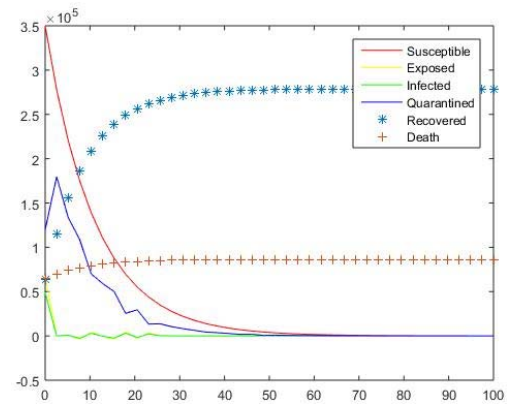
Param.	α	β	γ	δ	λ	n
Value	0.02	0.001	6	14	10^{-4}	10^{-6}

Figure 12 :Test13.



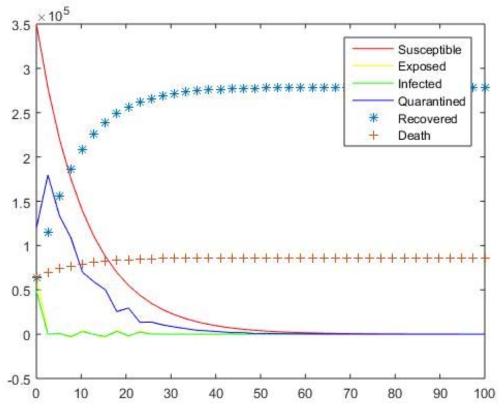
Param.	α	β	γ	δ	λ	n
Value	0.02	10^{-3}	6	14	10^{-6}	10^{-3}

Figure 13 : Test 14.



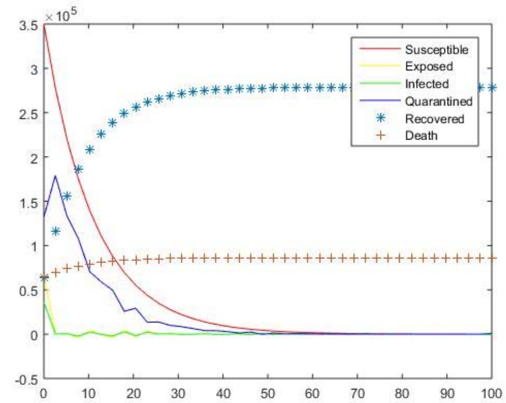
Param.	α	β	γ	δ	λ	n
Value	0.09	0.9	6	14	0.1	0.01

Figure 14 : Test 15.



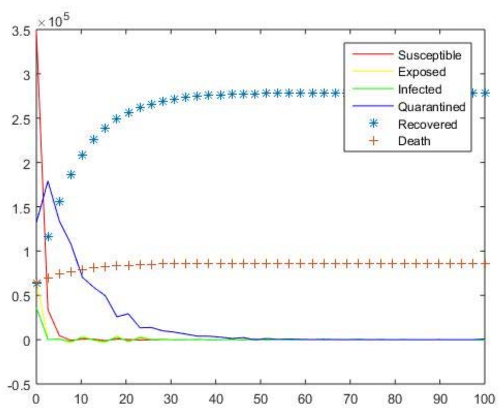
Param.	α	β	γ	δ	λ	n
Value	0.09	10^{-4}	6	14	0.1	0.01

Figure 15 : Test 16.



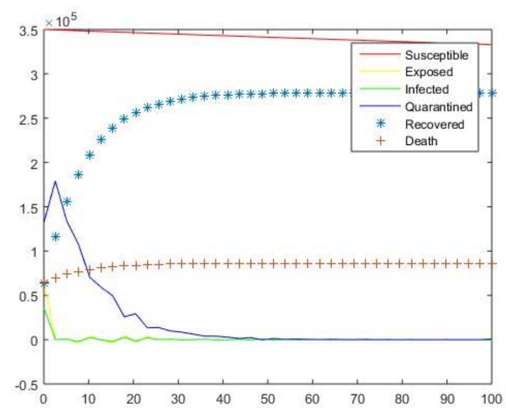
Param.	α	β	γ	δ	λ	n
Value	0.09	10^{-4}	6	21	0.1	0.01

Figure 16 : Test 17.



Param.	α	β	γ	δ	λ	n
Value	0.9	10^{-4}	6	21	0.1	0.01

Figure 17 : Test 18,



Param.	α	β	γ	δ	λ	n
Value	0.0005	10^{-4}	6	21	0.1	0.01

Figure 18 : Test 19.

6. Conclusion and Comments

When α is large, S decreases sharply, on the other hand if it is small S gradually decreases Test 1., Test18. and Test 19. The number of exposed and infected people decreases according to the quarantine time Test 1 and Test 2, when δ increases (if it goes from 14 to 25, we see that among the people exposed, some will not be infected. On the other hand, as the quarantine time decreases, we see that compartments E and I look the same. This allows us to say that a long period of quarantine would surely allow some to leave compartment I without need for treatment even if they have been in contact with infected people they may not be infected. When λ is large and β small, the number of people cured increases exponentially Test 5. and Test 6. On the other hand when κ is large, we obtain the opposite and in this case, from the 20*th*, the other compartments Q, E and I tend to disappear, that is to say that all the affected people are cured or died according to the rate β. Test 6. allows us to highlight the influence of the parameter β, the number of people quarantined decreases in favour of people who are cured and deceased and Test 7. shows that when the latent time is greater than the quarantine time, at the beginning the number of infected people is higher than the exposed people then will tend to disappear over time because λ and κ increase and at the same time the number of people put in quarantine decreases.

The numerical results obtained are encouraging compared to those existing in the literature. It would be interesting in the future to use the same techniques to study stochastic models for Covid-19. It would also be interesting to take into account certain other controls, such as vaccination.

Data Availability No data were used to support this paper.

Conflicts of Interest The authors declare no conflicts of interest with regard to any individual or organization.

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